

A Decentralized Charging Control of a Multiple-Receiver Wireless Power Transfer System Using Ultracapacitor Semi-active Topology

He Yin, Minfan Fu, Chen Zhao
Univ. of Michigan-Shanghai Jiao Tong
Univ. Joint Institute,
Shanghai Jiao Tong University,
Shanghai, P. R. China
Email: yyy@sjtu.edu.cn
fuminfan@sjtu.edu.cn
zhaochen815@aliyun.com

Chengbin Ma^{*1,2}
1. Univ. of Michigan-Shanghai Jiao Tong
Univ. Joint Institute,
2. School of Mechanical Engineering
Shanghai Jiao Tong University,
Shanghai, P. R. China
Email: chbma@sjtu.edu.cn

Abstract—A decentralized charging control of a multiple-receiver wireless power transfer (WPT) system is proposed and discussed in this paper. Different from charging battery alone systems through WPT technology, using an additional ultracapacitor (UC) combined with the battery alone system could both achieve the power requirement and a high efficiency. Then the charging control problem is converted into a two-stage Stackelberg game with the transmitter, receivers, and loads as players. The objectives of the receivers are to maintain the voltage of the UC pack while the objectives of the loads are to be charged at their preferred charging power. In the first stage, a Stackelberg and a Nash equilibrium are found between the transmitter and receivers. While, in the second stage, another Stackelberg equilibrium is found between the receivers and the loads. At each control instant, the equilibriums are reached through learning algorithm with KarushKuhnTucker conditions. The simulation results show that the game theoretic approach gives a flexible and efficient performance.

Index Terms—Wireless power transfer, game theory, ultracapacitor semi-active topology

I. INTRODUCTION

Thanks to the rising interests of the clean energy and the concerns of the global warming, batteries, especially the lithium batteries, are becoming more and more popular in daily life. In order to overcome the disadvantages of the batteries, i.e., short cycle life and low power density, the combination of the batteries and ultracapacitors (UCs), i.e., the battery-UC hybrid energy storage systems (HESSs) have been widely studied [1]–[3]. UCs have been discussed for over two decades and now they have already been applied in personal electronics, e.g., cell phones [4]. Therefore, in a medium power (i.e., 10 W to 40 W) wireless power transfer (WPT) system, the UCs can be treated as an energy buffer to realize fast charging rather than the traditional energy suppliers. This novel application for both UCs and battery-UC HESSs is analysed and discussed in this paper.

In a WPT system, the load is always an energy storage system while the battery alone system is the most commonly

used one. However, problems exist in this system that the WPT system can not both work at the most efficiency power level and satisfy the charging power requirement of the battery alone system because these two power levels may be quite different. The most common solution of this problem is to redesign the WPT system with the specific load power level. However, this solution allows only one specific power level for the load and lacks flexibility. On the other hand, using UC-battery HESS replacing the battery-alone system could be a better solution to satisfy both the high efficiency and the power requirement without redesigning the WPT system. The WPT system could remain turned on and work at the maximum efficiency point to charge the UCs while the batteries are charged from the UCs with their satisfied power level. The difference of the power level between their satisfied power level and transmitter power level can be absorbed by the UCs while the WPT system can be turned off when the energy level of the UCs is high enough. Besides charging a single receiver, the WPT system could supply energy to other receivers during the turn-off time. Through a so called time-division based charging strategy [5], multiple energy storage system can be charged simultaneously in a virtual manner. Therefore, a high overall efficiency and power requirement can be met with multiple-receiver WPT systems. Such time-division based control is discussed and applied in this paper.

As discussed in previous paragraph, control strategies play an essential role in the power distribution among the multiple-receivers in a WPT system. [6] discusses the ideal efficiency distribution for a multiple-receiver WPT system. However, a real-time control is still required for a real-world application. A time division based control is applied in a multiple-receiver WPT system using UCs [5]. The above discussions focus on a static case while these receivers may arrive or depart the system at any time. The charging control strategies discussed previously are all centralized controls which can not deal with these unpredictable cases. On the other hand, for similar rea-

sons, decentralized control strategies can satisfy these system requirement well. Game theory, as a famous decentralized control strategy, is a branch of applied mathematics that analyses the interactions and conflicts among players in a game (e.g., systems and environment.) [7]. Different from centralized control strategies, the game theoretic approach using only locate parameters and could achieve a flexible and reconfigurable solution. Such kind of controls have been applied instead of centralized control in micro grid and DC power system [8]–[10]. In this paper, in order to achieve a high system efficiency and flexibility, a two stage Stackelberg game is used to solve the power distribution problem in a multiple-receiver WPT system through using UC semi-active topologies.

This paper presents a game-theoretic control of the charging time distribution in a time-division based multiple-receiver WPT system with UC semi-active topology. The charging time distribution control problem is treated as a two-stage Stackelberg game while a Stackelberg equilibrium between the transmitter and receivers, a Nash equilibrium among receivers, and a Stackelberg equilibrium between the receiver and the load are determined at each control instant. The preferences of the transmitter, receivers, and loads are modeled through cost functions, maximizing the turn-off time, maintaining the voltage level of the UC pack, and charging at the most preferred power respectively. With several iterations, a stable state can be reached for each situation. A simulation under Matlab environment is built with two different case, i.e., arriving the game and departing the game, to verify the theoretic analysis and the performance of the game-theoretic control, i.e., high efficiency and flexibility.

II. SYSTEM CONFIGURATION AND MODELLING

A. Circuit Models

A general multiple-receiver WPT system topology is shown in Fig. 1. In this topology, the entire system contains one transmitter and N receivers. The transmitter has one direct current (DC)-power supply, one power amplifier (PA), and one transmitter coil. The DC-power supply here provides the input voltage of the PA, V_{pa} , which is the design as the upper limit value of the DC-power supply, 30 V. It is noticed that a higher V_{pa} means a higher output power of the PA. The PA used in this paper is a classic class-E PA with optimal impedance, 15 Ω (i.e., it has a highest efficiency when the impedance is 15 Ω). It transforms the DC power to alternating current (AC) power with a frequency, 6.78 MHz. Note that the class E PA is used here as an example for it high efficiency.

Each receiver, it contains one receiver coil, one rectifier, one DC-DC converter, and one UC semi-active HESS. The rectifier here is a classic full-wave rectifier while the DC-DC converter is a buck converter to control the impedance. The main purpose to tune the impedance is to reach the highest efficiency through the most efficiency point tracking control proposed in Section III-A. The UC semi-active HESSs are treated as the loads for the general WPT system.

The UC semi-active HESS consists of an UC pack, a DC-DC converter, a battery pack, and the load. Among the battery-UC hybrid energy storage systems, UC semi-active topologies are the most satisfied one because the current of the UC pack should be bi-directional and only one DC-DC converter is used for a high efficiency. Since the dynamic performance of the battery and UC pack are not focused here, they are simply modeled as shown in Fig. 2. On one hand, the benefits of using an UC semi-active topology in the WPT system is to provide a continuous and constant charging power to the battery pack. The UC pack plays as an energy buffer here. On the other hand, the UC semi-active topology also provides benefits for discharging the battery [11]. Note that the battery pack here can be replaced by any other kind of energy storage systems (ESSs). If the ESS could be charged with discontinuous power, the UC pack and the DC-DC converter in the UC semi-active topology can be removed. The DC-DC converter in the UC semi-active topology is a classic bi-directional buck-boost converter which is used to control the charging current. Since using UC semi-active topologies to provide power to the load has been discussed [11], the load is just used when the UC semi-active HESS is not charging which is not discussed in this paper.

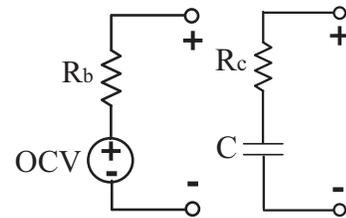


Fig. 2. The battery and UC models. (a) Battery pack model. (b) UC pack model.

III. NON-COOPERATIVE GENERALIZED STACKELBERG GAME

In this paper, a two-stage non-cooperative generalized Stackelberg game is set up, as shown in Fig. 3. In the first stage, the transmitter and the receivers will determine the charging time while in the second stage, the receiver and the load will determine the charging power.

A. Most Efficiency Point Tracking Control

As mentioned in the previous section, due to the benefits of the time division based control, the multiple-receiver WPT system is actually a single-receiver WPT system at any time instant. The relationship between the efficiency and the impedance of the entire system with a given mutual inductance is analysed and discussed in [12]. The relationship of the impedance, k (i.e., the coupling coefficient, only affected by the coil location.), and the system efficiency for the proposed test bench is shown in Fig. 4. It can be observed that with a given k , there only exists a single maximum efficiency point (i.e., the peak in the contour). A tracking technology for the single-receiver WPT system is discussed in [12]. In

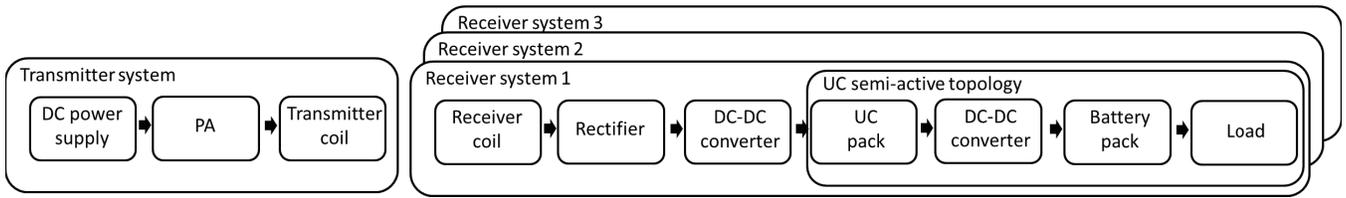


Fig. 1. The system topology.

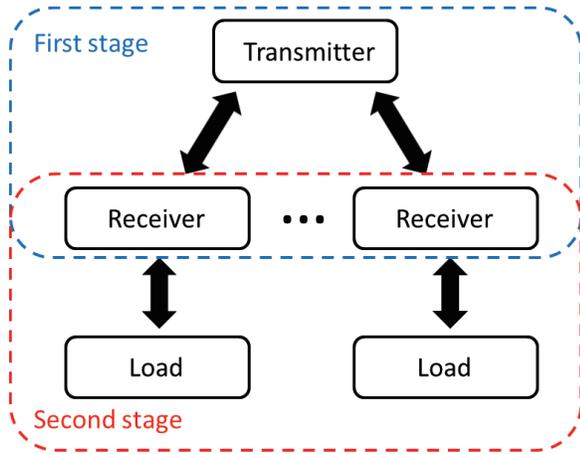


Fig. 3. The two-stage Stackelberg game.

order to increase the system efficiency, the most efficiency point tracking technology is applied at any time instant.

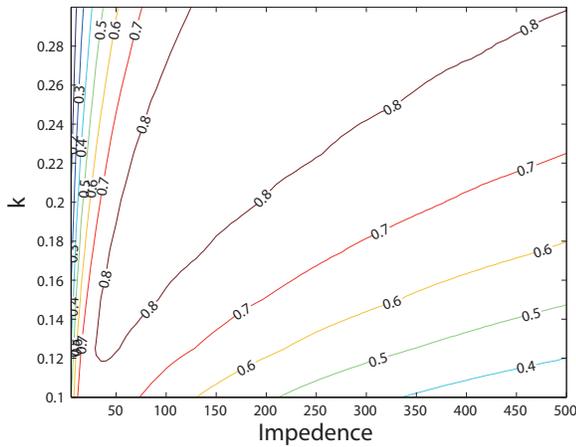


Fig. 4. The efficiency map of the WPT system.

B. First stage Stackelberg Game

In the first stage, the transmitter is the leader while the receivers are the followers. For the leader, the control variable is the minimum turn off time, x_{off} . For the followers, the strategies are the charging time x_i s. They will be charged in sequence, i.e., x_1, x_2, x_3, x_{off} . These strategies have a common constraint, $\sum x_i + x_{off} = T$, where T is the period of the control instant. The T should be limited by the capacitance of

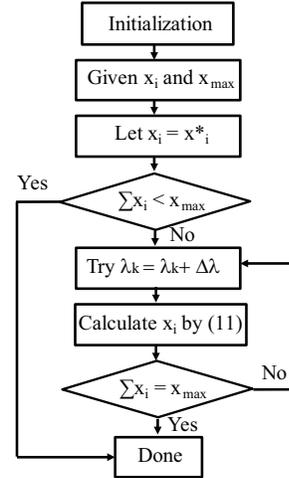


Fig. 5. The pseudo code for the learning algorithm.

the UC pack, i.e., $[\max(\frac{C(v_{max}^2 - v_{i,1}^2)}{2(P_i - P_{max,i})}), \min(\frac{C(v_{i,1}^2 - v_{min}^2)}{2(P_{max,i})})]$. According to the configuration parameters, the T can be selected as 60 s.

1) *Problem formulation:* In this game, both the leader and the followers have their objectives called cost functions here. They try to maximize the cost function with tuning their own strategies given the strategies of other players. On the follower side, the receivers try to keep the energy level of the UC packs at the initial state and the final state (after T .) to be as close as possible. This is because for a receiver, keeping the energy level of the UC pack could maintain this receiver working normally. Thus, the cost function of a follower can be shown as follows:

$$u_i = 1 - t_i(x_i - x'_i)^2, \quad (1)$$

$$t_i = \frac{1}{(x'_i)^2} \quad (2)$$

$$\sum x_i \leq T - x_{off}, \quad (3)$$

where x'_i is the charging time with which the UC pack could reach the initial energy level in the next turn. The x'_i is calculated as following:

$$x'_{i,k} = x'_{i,k-1} + \frac{C(v_{i,k-1}^2 - v_{i,k}^2)}{2(P_i - P_{max,i})}, \quad (4)$$

where P_i is the average charging power of the i th receiver and $P_{max,i}$ is the maximum charging power of the load. Since the capacitance of the UC is limited, the x_i should be

limited within $[0, \frac{C(v_{max}^2 - v_{i,1}^2)}{2(P_i - P_{max,i})}]$. This parameter is determined by calculating the additional charging time to make the voltage of the UC pack to be the initial one.

On the leader side, in order to avoid additional consumption, the transmitter tries to minimize the charging time, i.e., maximize x_{off} (if the $\sum x_i < T - x_{off}$, the $x_{off} = T - \sum x_i$). Thus, the cost function for the transmitter is shown as following:

$$v_i = x_{off}, \quad (5)$$

$$\sum x_{i,k+1} \geq \sum x_{i,k}, \quad (6)$$

$$0 \leq x_{off} \leq T. \quad (7)$$

2) *Generalized Nash equilibrium*: In order to solve the cost function as (1), traditional solving procedure for finding a Nash equilibrium is not suitable because the strategies of other followers only exist in the constrains rather than in the cost functions. Therefore, this problem is a generalized Nash equilibrium [13]. In this paper, KarushKuhnTucker (KKT) conditions are used to reach the generalized Nash equilibrium point which is more socially stable than other generalized Nash equilibrium solutions [13]. The utility function of each receiver is converted into a Lagrangian function:

$$L(x_i, \lambda_i) = u_i + \lambda_i G(x_i, \bar{x}_i), \quad (8)$$

$$G(x_i, \bar{x}_i) = \sum x_i + x_{off} - T, \quad (9)$$

$$0 \leq \lambda_i \perp -G(x_i, \bar{x}_i) \geq 0, \quad (10)$$

where λ_i s represent the Lagrange coefficients.

By taking derivative of x_i and take every λ_i to be equal:

$$\frac{dL_i}{dx_i} = -2t_i(x_i - x'_i) + \lambda_i = 0, \quad (11)$$

$$\lambda_1 = \lambda_2 = \dots = \lambda_N. \quad (12)$$

Then, the solutions can be found. (11) and (12) give two solutions:

$$x_i = x'_i, \quad (13)$$

and

$$x_i = \frac{ax_{max}}{\sum \frac{a}{t_k}} + x'_i - \frac{ax'_k}{\sum \frac{a}{t_k}}, \quad (14)$$

$$a = \prod t_i. \quad (15)$$

For the first kind of solution, the constraint (10) does not hold while for the second kind of solution, the constraint (10) holds. If the first kind of solution holds, the λ_i should be zero. The entire power requirement is less than that of the power supply. In this case, each follower could maintain the UC energy level to be the same as the initial value. IF the second kind of solution holds, λ_i should not be zero. In this case, the power requirement of the receivers are larger than that of the power supply. All receivers have to negotiate and reach a balance charging time distribution.

A learning algorithm to reach the generalized Nash equilibrium is required for the implementation of the decentralized

control in a real world controller. One of the possible solution is try to determine the λ_i starting from zero. The detail learning algorithm is shown in Fig. 5.

3) *Generalized Stackelberg equilibrium*: Given the previous solution set of the receivers, the leader could determine the x_{off} accordingly. Thus the strategy of the leader can be achieved by a simple rule-based strategy. If the followers have chosen the first kind of solution, the leader will reduce the charging time. On the other hand, if the followers have chosen the second kind of solution, the leader will increase the charging time. In order to determine which kind of solution the followers have decided, a simple rule-based method is shown as following:

$$\text{If } \sum x_i \geq \sum x_{i-1}, \quad (16)$$

$$x_{off} = x_{off} - \Delta x, \quad (17)$$

$$\text{else, } x_{off} = x_{off} + \Delta x, \quad (18)$$

where the Δx is determined as $5\%T$.

C. Second stage Stackelberg Game

In second-stage Stackelberg game, the receiver is the leader while the load is the follower. The objective of the leader is the same as in the first stage, i.e., to maintain the energy level of the UC pack as in the previous stage, the utility function is also the same as in the first stage. When the x_{off} comes into zero and the receivers still decides to choose the second kind of solution, the receiver could not both supply enough power to the load and maintain the energy level of the UC pack. Thus, in order to maintain the energy level of the UC pack, the upper level of the load, $P_{max,i}$ (control variable of the receiver.) should be lower. The solution of the receiver is determined:

$$\text{If } x_{off} = 0 \text{ and } v_{i,k-1} > v_{i,k}, \quad (19)$$

$$P_{max,i} = P_{max,i-1} - \Delta p, \quad (20)$$

$$\text{else, } P_{max,i} = P_{max,i-1} + \Delta p, \quad (21)$$

where i_{max} is the maximum charging current of the load and Δp is determined as $5\%P_{max,1}$. Similar as x_i , the upper limit of the $p_{l,i}$ is $P_{max,1}$ which will be introduced later.

On the other hand, the objective of the follower is to be charged as its most satisfied charging current, $\frac{C}{3}$. The $P_{max,1}$ is equal to this value timing the voltage of the load. The utility function of the load is shown as,

$$v_R = p_{l,i} \quad (22)$$

$$p_{l,i} \leq P_{max,i}, \quad (23)$$

where $p_{l,i}$ means the power absorbed by the load. The solution of this utility function is quite simple that $p_{l,i} = P_{max,i}$.

IV. SIMULATION RESULTS

A. Case Study

1) *Case One*: Two different cases, i.e., receivers arrives the game and receivers departing the game are discussed and analysis here to show the performance of the game theory

based control facing unexpected future. In case one, there are three receivers in the game at the initial state. Then they depart the game one by one at different time instant. As shown in Fig. 6, in the first ten T s, there are three receivers while the transmitter and receivers reach an equilibrium through several iterations. As shown in Fig. 7, since the P_{dc} can not support the entire charging powers (i.e., $\frac{C}{3}$), the load current also converges to a lower value. Then, in the next ten T s, one receiver departs the game, a new equilibrium is reached. A similar result can be reached in the last T s. As shown in Fig. 8, the voltage map of the UC pack clearly shows that the terminal voltage after average T also converges to the initial state at each condition. A more detailed voltage map is shown in Fig. 9, through which relationship between the UC voltage map in Fig. 8 and the charging time distribution in Fig. 6 is shown. In order to show the performance of the efficiency in the proposed control, figure. 10 shows the efficiency in a single period with three different receivers. It can be concluded that over 62 % system efficiency can be reached from the transmitter to the UC pack.

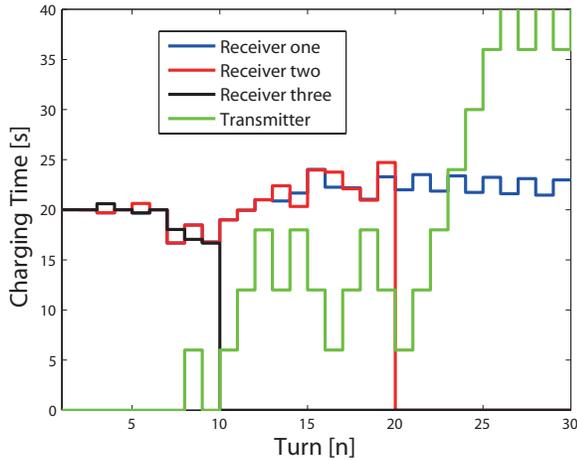


Fig. 6. The charging time distribution.

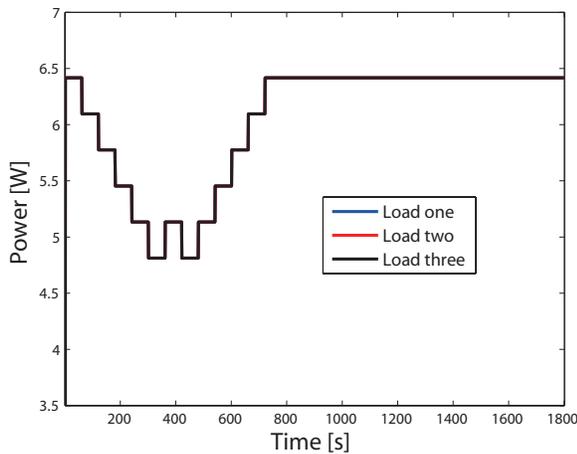


Fig. 7. The load current map.

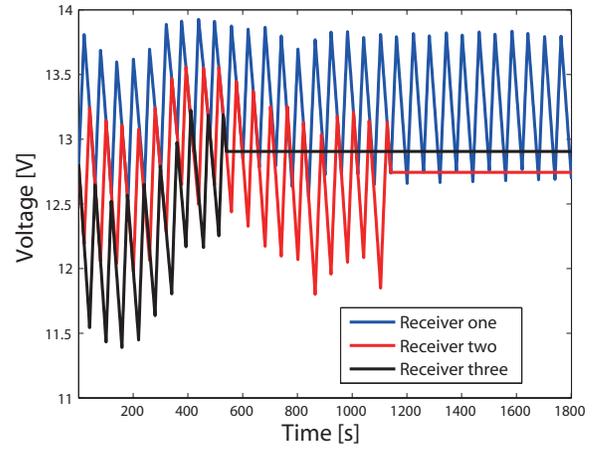


Fig. 8. The UC voltage map.

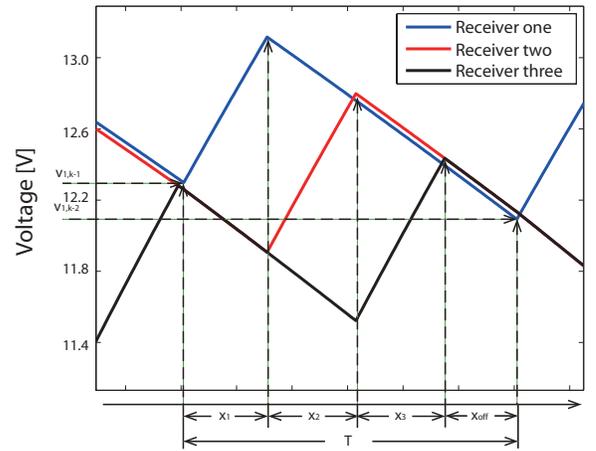


Fig. 9. The relationship between the voltage map and the charging time distribution.

2) *Case Two*: In this case, only one receiver exists at the initial state, and then another two receivers arrive the game one by one. As shown in Fig. 11, Fig. 12 and Fig. 13, similar results can be reached with a arriving game. Through showing these two cases, the a flexible control performance can be proved.

V. CONCLUSIONS

This paper proposes a game theory based decentralized charging control of a multiple-receiver wireless power transfer system with ultracapacitor semi-active topology. The charging control problem is converted into a two-stage noncooperative Stackelberg game. It is solved by reaching a Stackelberg equilibrium between the transmitter and receivers, a Nash equilibrium among receivers, and a Stackelberg equilibrium between the receiver and the load at each control instant. The objectives of the transmitter, receivers, and loads are modeled by different cost functions, which aim to maximize the turn-off time, maintain the voltage level of the UC pack, and be charged at the most preferred power respectively. A simulation under Matlab environment is built with two different case, i.e.,

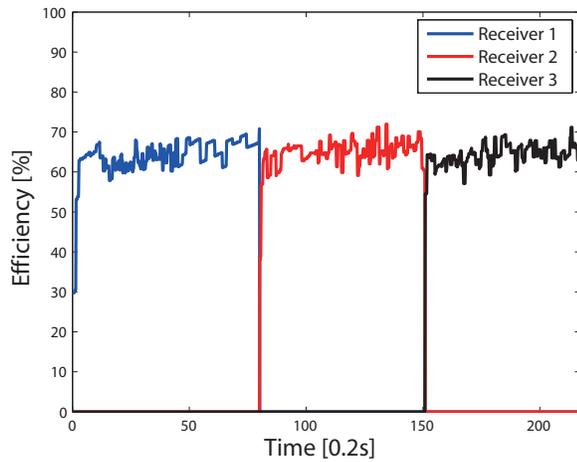


Fig. 10. The efficiency map for a single period.

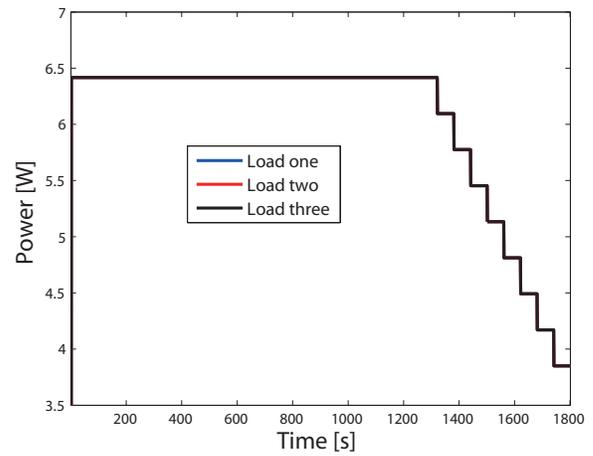


Fig. 12. The load current map.

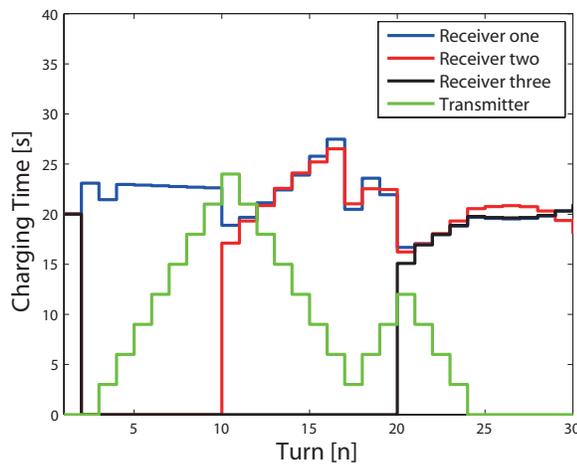


Fig. 11. The charging time distribution.

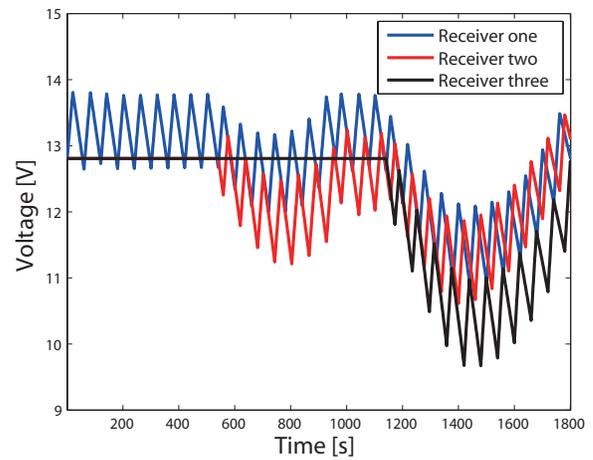


Fig. 13. The UC voltage map.

arriving and departing the game, to verify the theoretic analysis and to show the performance of the proposed game-theoretic control, i.e., high efficiency and flexibility.

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